1989

1a. $0 = x^3 - 7x + 6$

$$\begin{array}{r|rrrr} \checkmark\ 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2 + x - 6 = 0$ (3)

$(x+3)(x-2) = 0$

$x=1 \quad x=-3 \quad x=2$ (or graph the function to find zeros)

b. $f' = 3x^2 - 7$ (1)

$f'(-1) = 3(-1)^2 - 7$

$= 3 - 7 = -4$

we need y at $x = -1$

$f(-1) = (-1)^3 - 7(-1) + 6 = 12$ (1)

$\therefore -4 = \dfrac{y-12}{x+1}$

$-4(x+1) = y - 12$ (1) or $y = -4x + 8$

c. $f'(c) = \dfrac{f(3) - f(1)}{3-1}$ (1)

$3c^2 - 7 = \dfrac{3^3 - 7(3) + 6 - [1^3 - 7(1) + 6]}{2}$ (1)

$3c^2 - 7 = 6$

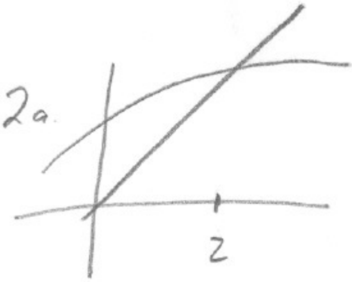
$3c^2 = 13$

$c^2 = \dfrac{13}{3}$

$c = +\sqrt{\dfrac{13}{3}}$ (1)

note: $-\sqrt{\dfrac{13}{3}}$ does not fall in the interval

2a.



(1) (1)

$\text{Area} = \int_0^2 \left(\sqrt{6x-4} - 2x\right)dx$

$= 2.222$ sq units (1)

(3)

b. $V = \int_0^2 \left[\pi\left(\sqrt{6x+4}\right)^2 - \pi(2x)^2\right]dx$

(3)

c. $V = \int_0^2 2\pi x\left[\sqrt{6x+4} - 2x\right]dx$

3a. $a = 4\cos 2t$

$V = \int 4\cos 2t$ (1)

$V = 2\sin 2t + c$

$V(0) = 1$ (1)

$1 = 2\sin(0) + c$

$c = 1$

$\therefore V(t) = 2\sin 2t + 1$ (1)

b. $x = \int 2\sin 2t + 1$ (1)

$x = -\cos 2t + t + c$

$x(0) = 0$ (1)

$0 = -\cos 2(0) + 0 + c$

$c = 1$

$x(t) = t - \cos 2t + 1$ (1)

3c. At rest when $v=0$

$2\sin 2t+1=0$ (1)

$\sin 2t = -\frac{1}{2}$

GRAPH TO FIND SOLUTIONS

$t=1.833, \ 2.8798$ (2)

$\frac{7\pi}{12} \quad \frac{11\pi}{12}$

4a. $x^2-4>0$

$x^2>4$

$x>2$ or $x<-2$ (2)

$|x|>2$

b. $x^2-4=0$ (1)

$x=\pm 2$ Vertical

c. $\lim_{x\to+\infty} \frac{x}{\sqrt{x^2-4}} = \frac{\infty}{\sqrt{\infty^2}} = 1$ (1)

$y=1$

$\lim_{x\to-\infty} \frac{-\infty}{\sqrt{(-\infty)^2}} = -1$ (1)

$y=-1$

d. (3) $f' = \dfrac{\sqrt{x^2-4}\,(1) - x\cdot\frac{1}{2}(x^2-4)^{-\frac{1}{2}}(2x)}{\left(\sqrt{x^2-4}\right)^2}$

5a. Horizontal tangent when $m=0$

$f'(x)=0$ at $x=-7,-1,4,8$ (3)

b. $x=-1$ and $x=8$ (2)

f' goes from $+$ to 0 to $-$ (1)

c. $(-3,2)$ and $(6,10)$ (3)

6a. $\frac{dy}{dt}=ky$

$\frac{dy}{y}=k\,dt$ (1)

$\ln y = kt+c$

$e^{\ln y} = e^{kt+c}$

$y=Ce^{kt}$ (1)

$t=0 \quad y=1{,}000{,}000$

$1{,}000{,}000 = Ce^{0}$

$C=1{,}000{,}000$

$y=1{,}000{,}000\,e^{kt}$ (1)

$500{,}000 = 1{,}000{,}000\,e^{k(6)}$

$\frac{1}{2} = e^{6k}$

$\ln\frac{1}{2} = 6k$

$k=-.1155245$ (1)

$y=1{,}000{,}000\,e^{-.1155245t}$ (1)

b. $\frac{dy}{dt}=ky$ (2)

$\frac{dy}{dt} = -.1155245\,(600{,}000)$

$= -69314.718$

c. $50{,}000 = 1{,}000{,}000\,e^{-.1155245t}$

$\frac{1}{20} = e^{-.1155245t}$

$\ln\frac{1}{20} = -.1155245t$ (2)

$\dfrac{\ln\frac{1}{20}}{-.1155245} = 25.932 = t$